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Also that the value of an assurance payable at the instant of the death of the first of the two joint lives x and y, or $\overline{\mathrm{A}}_{xy}$, is, on the same supposition,

$$2\frac{i\delta-i+\delta}{(1+i)\delta^2}+2\frac{2i\delta-2i+2\delta-i^2}{(1+i)\delta^2}a_{xy}+\frac{2i-2\delta-i\delta}{\delta^2}\left(\frac{a_{x-1\cdot y}}{p_{x-1}}+\frac{a_{x\cdot y-1}}{p_{y-1}}\right).\ .\ .\ (21)$$

On the Value of Reversionary Life Interests. By the late GRIFFITH DAVIES, ESQ., F.R.S. *Communicated by* GRIFFITH DAVIES, ESQ., *Actuary of the Law Life Assurance Society.*

TO find the present value of a Survivorship Annuity of £1 on the life of A to be entered on at the death of B, supposing the purchaser to effect an increasing insurance on A's life so that the sum insured may correspond with the amount of his outlay (of purchase money, premiums, and interest) at the end of each year up to the average duration of the joint lives of A and B, and to be at the end of that period of the exact amount which the annuity purchased would cover the annual interest thereon, beside providing for the premiums to keep it insured during the remainder of A's life.

Let S = sum advanced as purchase money for proposed annuity.
P = equated annual premium for requisite insurance.
p = annual premium for the insurance of £1 on A's life.
e = expectation of joint lives.
i = rate of interest allowed the purchaser.
$r=1+i$ = amount of £1.
$v=\frac{1}{r}$ = present worth of £1.
$d=1-v$.

D, N, M, &c., as in Table XIII. (Davies' "Tables of Life Contingencies," published in 1825).

Then it is manifest that the improved amount of the purchase money at the end of x years $=\mathrm{S}r^x$; the improved amount of the premiums at the end of the same period $=\frac{\mathrm{P}r(r^x-1)}{i}$, and consequently the sum of both which then constitutes the aggregate outlay of the purchaser $=\mathrm{S}r^x+\frac{\mathrm{P}r(r^x-1)}{i}$, and if in this expression we substitute e for x we have the amount of the purchaser's outlay at the end of the period which A and B may be expected to live together $=Sr^e+\frac{Pr(r^e-1)}{i}$, the annual interest on which $=Sir^e+Pr^{e+1}-Pr$, and if to this we add the annual premium P we have $Sir^e+Pr^{e+1}-Pr+P=Sir^e+Pr^{e+1}-Pi=1$ by the conditions of the problem.

$$\therefore\quad P=\frac{1-Sir^e}{r^{e+1}-i}=\frac{v^e-Si}{r-iv^e}$$

Referring again to the foregoing expression, we have the amount of the purchaser's outlay

at the end of x years . . . $Sr^x+\frac{Pr(r^x-1)}{i}$

at the end of $(x-1)$ years . $Sr^{x-1}+\frac{Pr(r^{x-1}-1)}{i}$

The increment during the xth year $=Sr^{x-1}(r-1)+\frac{Pr^x(r-1)}{i}$ $=Sir^{x-1}+Pr^x$, and if in this expression we make x successively $=1, 2, 3, \ldots e$ we find that

the increment during the 1st year $=Si+Pr$

Do. 2nd „ $=(Si+Pr)r$

Do. 3rd „ $=(Si+Pr)r^2$

. . . .

Do. eth „ $=(Si+Pr)r^{e-1}$

Hence assuming that the sum assured cannot be claimed from the Insurance Office until the end of the year in which A may die it is manifest, in order to satisfy the conditions of the problem, the following insurances must be effected, viz.: — the purchase money and the first year's increment as an immediate insurance on A's life; and the second year's increment as a further insurance on the same life deferred for one year, the third year's as a still further insurance deferred for two years—and so on to the average duration of the joint lives of A and B.

It therefore follows, from the construction of the Table referred to, that the single premium for the

1st of the said insurances $=\frac{M}{D}\{S+(Si+Pr)\}$

2nd do. $=\frac{{}^1Mr}{D}\{Si+Pr\}$

3rd do. $=\frac{{}^2Mr^2}{D}\{Si+Pr\}$

. . . .

eth do. $=\frac{{}^{e-1}Mr^{e-1}}{D}\{Si+Pr\}$

The sum of which

$$=\frac{SM}{D}+\frac{Si+Pr}{D}(M+{}^{1}Mr+{}^{2}Mr^{2}+\ \ldots\ldots\ {}^{e-1}Mr^{e-1})$$

and dividing this sum by $(1+A)$ or its equivalent $\frac{{}_{1}N}{D}$ we have the equated annual premium P.

$$P=\frac{SM}{{}_{1}N}+\frac{Si+Pr}{{}_{1}N}(M+{}^{1}Mr+{}^{2}Mr^{2}+\ \ldots\ {}^{e-1}Mr^{e-1})$$

But $\frac{M}{{}_{1}N}=$ annual premium for the assurance of £1, which put $=p$; and if we put $\frac{1}{{}_{1}N}(M+{}^{1}Mr+{}^{2}Mr^{2}+\ \ldots\ \&c.)=Q$, leaving the method of determining the value of Q to be shown hereafter, we have

$$P=Sp+SiQ+PrQ,\ \text{or}\ P=\frac{S(p+iQ)}{1-rQ}.$$

Hence equating this expression with the value of P already determined in terms of S, we have

$$\frac{S(p+iQ)}{1-rQ}=\frac{v^{e}-Si}{r-iv^{e}},\ \text{from which,}$$

$$S=\frac{v-Q}{p(r^{e}-d)+d(r^{e}-iQ)},\ \text{and}\ P=\frac{v(p+iQ)}{p(r^{e}-d)+d(r^{e}-iQ)}.$$

Having now determined the values of S and P in terms of Q and known quantities, it only remains to point out the method of finding the value of Q. For this purpose, let n represent the age of the life A, on which the annuity depends; and let

$$L=Mr^{n}+{}^{1}Mr^{n+1}+{}^{2}Mr^{n+2}+\ \&c.$$
$${}^{e}L={}^{e}Mr^{n+e}+{}^{e+1}Mr^{n+e+1}+\ \&c.$$
$$\text{and}\quad K={}_{1}Nr^{n},\ \text{then}\ Q=\frac{L-{}^{e}L}{K}.$$

The annexed Table has been constructed from these values of L and K.

The foregoing is a copy of an investigation made, at least 40 years ago, by the late Mr. Griffith Davies; and the formula was used by him in cases relating to Survivorship Annuities.

Survivorship Annuities, &c., to pay 5 per Cent beside the Premium charged for Assurances by the Northampton 3 per Cent.

Age.	K.	L.	Age.	K.	L.	Age.	K.	L.
14	149532·0	263082·7	42	120195·3	162521·2	70	36615·9	40158·6
15	149486·3	260274·0	43	118005·4	158212·7	71	33475·5	36490·8
16	149362·5	257391·6	44	115733·9	153867·1	72	30410·8	32953·0
17	149156·4	254433·1	45	113380·9	149486·0	73	27436·4	29557·8
18	148868·0	251400·2	46	110946·9	145071·2	74	24567·7	26318·5
19	148500·8	248297·6	47	108432·1	140624·6	75	21822·0	23249·2
20	148059·3	245130·7	48	105837·5	136148·2	76	19217·3	20364·9
21	147545·9	241903·8	49	103163·4	131644·3	77	16772·8	17681·4
22	146966·6	238623·5	50	100411·4	127115·4	78	14495·5	15202·3
23	146323·9	235293·6	51	97585·0	122566·7	79	12387·3	12926·4
24	145616·6	231914·0	52	94691·6	118006·5	80	10444·8	10847·4
25	144842·9	228484·4	53	91736·6	113441·2	81	8673·4	8967·2
26	144001·7	225004·6	54	88723·4	108874·9	82	7083·0	7292·1
27	143091·0	221474·4	55	85655·2	104311·9	83	5678·6	5823·7
28	142109·8	217893·8	56	82535·5	99756·9	84	4465·3	4563·1
29	141056·6	214262·7	57	79369·0	95214·9	85	3452·6	3516·1
30	139929·9	210581·0	58	76160·4	90691·2	86	2623·9	2662·9
31	138728·1	206848·8	59	72914·6	86191·5	87	1959·0	1981·4
32	137449·9	203066·1	60	69637·2	81721·8	88	1436·1	1447·0
33	136094·2	199233·1	61	66334·6	77288·6	89	1034·5	1037·9
34	134659·5	195349·9	62	63013·4	72898·8	90	725·7	724·7
35	133144·3	191416·7	63	59680·9	68559·7	91	489·2	486·2
36	131547·7	187433·9	64	56341·4	64275·7	92	310·1	304·8
37	129868·3	183401·8	65	53003·5	60054·9	93	175·9	173·0
38	128105·0	179320·9	66	49672·5	55902·3	94	84·2	82·5
39	126256·8	175191·7	67	46357·5	51826·7	95	30·9	30·1
40	124322·8	171014·8	68	43069·0	47837·5	96	6·3	6·1
41	122301·8	166791·0	69	39817·9	43944·6			

On a Method of obtaining De Moivre's formula in the simplest terms. By M. CHARLON, *Manager of the "Spanish Phœnix," Madrid. Communicated by* M. HENRIQUEZ PIMENTEL, *Professor of Mathematics at The Hague, Holland.**

WHEN we consider in a table of mortality the consecutive terms which express the number living at each age, we remark that the differences between those numbers are almost constant in the middle ages of life; they change afterwards, remaining however uniform for another series of terms; so that the table can be considered as a series of decreasing arithmetical progressions. Let c be the difference between l_x and l_{x+1}; and let us suppose this difference constant for n years. We shall then have for the value of the life annuity during that time

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* The communication of M. Pimentel is written in French; but we have thought it more useful to give a translation. We have also altered the notation in some respects. —ED. *J. I. A.*